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Estimates of the earth's liquid outer core velocity using toroidal assumption and the DGRF/IGRF 1990-2000 magnetic models

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Abstract

This study aims to examine mechanisms responsible for the generation of the Earth's magnetic field and its corresponding secular variation induced by geodynamo processes within a perfectly conducting liquid outer core of the rotating Earth. It has been believed that the Earth's core is the primary source of geomagnetic production in a framework of the frozen-flux hypothesis. In time scales of order several years, we calculate the fluid's core velocity using a purely toroidal assumption by neglecting poloidal components. The toroidal flow is a two dimensional velocity adopted here to estimate the horizontal speed of the Earth's fluid outer core. Using magnetic models of the DGRF/IGRF 1990, 1995, 2000, numerical computation based on the assumption showed that the horizontal speed of the Earth's core was calculated in the range of 6.9–16.6 km/y, consistent with previous work. Estimates based on the radial field to determine the relative contributions of dipole and quadrupole components confirmed that the dipole dominates over the quadrupoles, hence the main part of the field, which is of fundamentally internal origin.

Keywords geodynamo; liquid outer core; secular variation; toroidal assumption

1. Introduction

The generation of the Earth's magnetic field and its corresponding secular variation (SV) is induced by geodynamo processes, which occur within an electrically conducting liquid outer core of the dynamic, rotating Earth (e.g., Glatzmaier & Roberts, 1995; Buffett, 2000; Prastowo, 2001). It has been widely believed that the Earth's fluid outer core is the primary source of geomagnetic production, for which the outer core is called the internal source (Roberts and Soward, 1992). The frozen-flux hypothesis (Roberts and Scott, 1965) is used to govern magnetic induction equation that relates the core dynamics to the geomagnetic field and the SV. For a relatively short time scale of order several years, the core velocity can then be estimated (Bloxham & Jackson, 1991; Prastowo & Bijaksana, 2002) using the radial component of the magnetic induction equation and the continuity equation near the core-mantle boundary (CMB). In this time scale, fluid flow near the CMB is assumed to be purely toroidal (Whaler, 1980, 1982) by neglecting all poloidal components associated with convective motions inside the core (Glatzmaier, 2001). The toroidal flow is a two dimensional velocity and is then utilised here to compute the horizontal speed of the fluid outer core.

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Research regarding variations in the geomagnetic field is beneficial to various disciplines in a broad sense, including educational sectors due to its potential to reduce misconception in physics at schools and universities, disaster risk reduction programs due to its possible use of designing a tsunami early warning system based on tsunami-generated magnetic signals, and its application to navigation systems in particular for safety issues in sea and air transportation.

In this study, we therefore examine mechanisms responsible for the production of the time-dependent geomagnetic field. The purposes in the present study are thus twofold. Firstly, we report on new numerical calculations using the 8–12th Generations of the DGRF/IGRF 1990, 1995, 2000 magnetic models to determine the outer core's velocity. Secondly, we then examine the current results in the context of previous work (Glatzmaier & Roberts, 1995; Buffett, 2000; Prastowo & Bijaksana, 2002) and whether there is a dependence of the results on the models used, with the aim of building a more comprehensive understanding of the outer core dynamics. All the results and discussions are based on the dipole approximation. Analytical arguments are briefly provided to support the dipole field as the major contributor to the Earth's magnetic field, which is of internal origin.

2. Data and methods

The data in the current study were the so-called Gauss geomagnetic coefficients written as $\{g_l^m\ ,\ h_l^m\}$ where $l \geq 1$ and $m \geq 0$ from the 8–12th Generations of the DGRF/IGRF 1990-2000 magnetic models released by International Association of Geomagnetism and Aeronomy (IAGA), Working Group Division V accessible at http://www.ngdc.noaa.gov/IAGA/vmod/ in the form of a series of numerical models for the main geomagnetic field of internal origin as it varies with time. The model utilized a truncated spherical harmonic function to represent the radial component B_r of the Earth's magnetic field (measured in nano tesla) as follows,

$$B_\mathrm{r} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} (l+1) \left(\frac{\mathrm{a}}{\mathrm{c}}\right)^{l+2} \{g_l^m(t) \cos m\varphi + h_l^m(t) \sin m\varphi\}\, P_l^m(\cos\theta), \quad (1)$$

where a and c denote the radii of the Earth's surface and the Earth's outer core, respectively, φ is the longitude, $P_l^m(\cos\theta)$ represent the Schmidt semi-normalized associated Legendre polynoms of degree l and order m, where θ is the co-latitude.

As noted earlier, we used dipole approximation for the main field of internal origin and thus the coefficients $\{g_l^m\ ,\ h_l^m\}$ in Eq. (1) are $g_1^0,\ g_1^1,\ h_1^1$ where g_1^0 is the axial term of the dipole and $g_1^1,\ h_1^1$ are the equatorial terms of the dipole. Table 1 below provides the first three-terms of a spherical harmonic series of the geomagnetic coefficients for three different periods of time, taken from the 12th Generation of the DGRF 1990–2000 magnetic models for example.

Table 1. A set of typical values of geomagnetic coefficients (in nano tesla) for the dipole case.

g/h	ℓ	m	DGRF-1990	DGRF-1995	DGRF-2000
g	1	0	–29775	–29692	–29619.4
g	1	1	–1848	–1784	–1728.2
h	1	1	5406	5306	5186.1

The radial field B_r in Eq. (1) was inserted into the radial component of the magnetic induction equation as follows,

$$\partial_\mathrm{t} B_\mathrm{r} + B_\mathrm{r}\, \nabla_\mathrm{h} . \mathbf{u_h} + \mathbf{u_h} . \nabla_\mathrm{h} B_\mathrm{r} = 0, \quad (2)$$

where the first term denotes the SV, $\mathbf{u_h}$ represents the horizontal speed at the core surface, and ∇_h is the horizontal derivative operator. Boundary conditions were applied near the CMB, causing the second term in Eq. (2) to vanish as no radial component of the flow velocity was assumed to penetrate out of the region of interest (e.g., Roberts & Soward, 1992).

Following Bloxham & Jackson (1991), we decomposed the horizontal flow speed $\mathbf{u_h}$ of the fluid outer core as two independent components, namely, poloidal and toroidal components, such that $\mathbf{u_h} = \mathbf{u_P} + \mathbf{u_T}$. In the context of a purely toroidal assumption (Whaler, 1982) adopted here, where no poloidal components of the flow exist, the horizontal flow speed was then simply written as $\mathbf{u_h} = \mathbf{u_T}$ where the toroidal component was defined as

$$\mathbf{u_T} = \nabla_h \times (\mathbf{r}\mathrm{T}) = \left(0,\ \frac{1}{\sin\theta}\frac{\partial \mathrm{T}}{\partial \varphi},\ -\frac{\partial \mathrm{T}}{\partial \theta}\right). \quad (3)$$

In Eq. (3), $\mathbf{r}$ is position in the Earth's spherical coordinates and T denotes a scalar function, assosiated with a purely toroidal flow, which can be expanded into a series of spherical harmonic terms of the Schmidt semi-normalized associated Legendre polynoms as follows,

$$\mathrm{T}(\theta,\varphi) = \sum_{l=1}^{\infty}\sum_{m=0}^{l}\left({}_{c}t_l^m \cos m\varphi + {}_{s}t_l^m \sin m\varphi\right) P_l^m(\cos\theta), \quad (4)$$

where t_l^m are toroidal coefficients of the flow. Eq. (4) was then solved for these coefficients using an invertion equation in the form of matrices for the flow as follows,

$$t_l^m = \mathrm{E}^{-1}\dot{g}, \quad (5)$$

where $\dot{g} = \{\dot{g}_l^m,\ \dot{h}_l^m\}$ are, following Prastowo (2001), the SV given by the corresponding models listed in Table 1 and E^{-1} is the inverse of Elsasser matrix (not detailed here). Table 2 below provides the first three-terms of the SV for three different periods of time, for example, taken from the 12th Generation of the DGRF 1990–2000 magnetic models.

Table 2. A set of typical values of secular variation (in nano tesla per year) for the dipole case.

$\dot{g}/\dot{h}$	ℓ	m	DGRF-1990	DGRF-1995	DGRF-2000
$\dot{g}$	1	0	16.6	14.5	13.0
$\dot{g}$	1	1	12.8	11.2	11.8
$\dot{h}$	1	1	–20.0	–24.0	–21.6

Thus, it is clear from Eqs. (1)–(5) that we computed the horizontal speed of the flow near the core surface using an inversion method. For a given period of time, say the DGRF-2000, estimates of the core speed require the Gauss coefficients in Table 1 and the corresponding SV in Table 2, where both are measured at the Earth's surface. By definition, the method was utilized to examine the core dynamics and its associated speed using the main field of internal origin and the SV it produced. Fig. 1 below is a simple diagram, showing calculation of the horizontal velocity of the Earth's outer core on the basis of toroidal assumption (Whaler 1980; 1982).

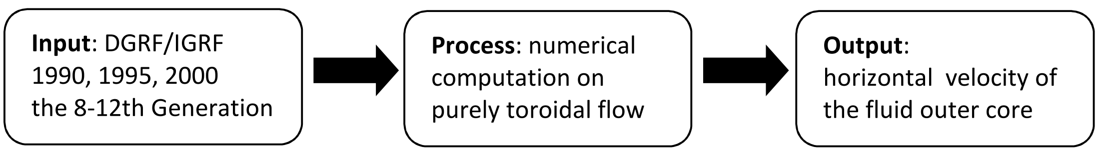

Figure 1. Simple diagram, showing numerical techniques for computing the core's horizontal speed in this study.

3. Results and discussions

As previously mentioned, in the process of examining physical mechanisms responsible for the generation of the geomagnetic field of internal origin inside the core and its associated time-varying main field in a relatively short period of time of order several years, we calculated the horizontal speed of the outer core based on toroidal flow assumption (Whaler, 1980; 1982) and the dipole approximation using the data from the DGRF/IGRF 1990-2000 magnetic models for various generations. Below are, for example, a series of numerical computation of the core speed for the dipole field using the 12th Generation of the DGRF 1990, 1995, 2000 magnetic models. Elsasser matrices and their corresponding reverses required in Eq. (5) are as follows,

$$\mathrm{E}_1 = \begin{pmatrix} 0 & 0.063468 & 0.021696 \\ -0.126937 & 0 & -0.699139 \\ -0.043392 & 0.699139 & 0 \end{pmatrix},$$

$$\mathrm{E}_1^{-1} = 10^7 \begin{pmatrix} -3.2224 & -0.1000 & 0.2925 \\ -0.2000 & -0.0062 & 0.0182 \\ 0.5851 & 0.0182 & -0.0531 \end{pmatrix},$$

$$\mathrm{E}_2 = \begin{pmatrix} 0 & 0.062435 & 0.021004 \\ -0.124871 & 0 & -0.696955 \\ -0.042007 & 0.696955 & 0 \end{pmatrix},$$

$$\mathrm{E}_2^{-1} = 10^6 \begin{pmatrix} -8.3529 & -0.2517 & 0.7483 \\ -0.5034 & -0.0152 & 0.0451 \\ 1.4966 & 0.0451 & -0.1341 \end{pmatrix},$$

$$\mathrm{E}_3 = \begin{pmatrix} 0 & 0.060886 & 0.020287 \\ -0.121771 & 0 & -0.695382 \\ -0.040575 & 0.695382 & 0 \end{pmatrix},$$

$$\mathrm{E}_3^{-1} = 10^6 \begin{pmatrix} -8.5667 & 0.2499 & -0.7501 \\ 0.4999 & 0.0146 & -0.0438 \\ -1.5001 & -0.0438 & -0.1341 \end{pmatrix},$$

where 1, 2, and 3 indices correspond to each model: the DGRF-1990, DGRF-1995, and DGRF-2000 used in the present study. Each of the inverses of Elsasser matrices above was then inserted into Eq. (5) combined with the SV data from Table 2. Thus, we could compute toroidal coefficients t_l^m for each proposed model using Eq. (5) as follows,

$$\begin{pmatrix} {}_{c}t_1^0 \\ {}_{c}t_1^1 \\ {}_{s}t_1^1 \end{pmatrix} = 10^7 \begin{pmatrix} -3.2224 & -0.1000 & 0.2925 \\ -0.2000 & -0.0062 & 0.0182 \\ 0.5851 & 0.0182 & -0.0531 \end{pmatrix} \begin{pmatrix} 16.6 \\ 12.8 \\ -20.0 \end{pmatrix} = 10^{-1} \begin{pmatrix} -6.0628 \\ -0.3763 \\ 1.1007 \end{pmatrix}$$

$$\begin{pmatrix} {}_{c}t_1^0 \\ {}_{c}t_1^1 \\ {}_{s}t_1^1 \end{pmatrix} = 10^6 \begin{pmatrix} -8.3529 & -0.2517 & 0.7483 \\ -0.5034 & -0.0152 & 0.0451 \\ 1.4966 & 0.0451 & -0.1341 \end{pmatrix} \begin{pmatrix} 14.5 \\ 11.2 \\ -24.0 \end{pmatrix} = 10^{-1} \begin{pmatrix} -1.4204 \\ -0.0856 \\ 0.2545 \end{pmatrix}$$

$$\begin{pmatrix} {}_{c}t_1^0 \\ {}_{c}t_1^1 \\ {}_{s}t_1^1 \end{pmatrix} = 10^6 \begin{pmatrix} -8.5667 & 0.2499 & -0.7501 \\ 0.4999 & 0.0146 & -0.0438 \\ -1.5001 & -0.0438 & -0.1341 \end{pmatrix} \begin{pmatrix} 13.0 \\ 11.8 \\ -21.6 \end{pmatrix} = 10^{-1} \begin{pmatrix} 1.3052 \\ 0.0762 \\ -0.2286 \end{pmatrix}$$

The above values for ${}_{c}t_1^0$, ${}_{c}t_1^1$, and ${}_{s}t_1^1$ for each epoch considered in the models were required to calculate the scalar function T in Eq. (4), after which the final step of speed calculations was then performed using Eq. (3). In this way, all these steps of the numerical computation were solved for the toroidal component $\mathbf{u_T}$ of the core velocity in Eq. (3). In the light of the purely toroidal flow, firstly proposed by Whaler (1980), we equalized $\mathbf{u_T}$ to $\mathbf{u_h}$ denoting the horizontal flow speed of the liquid outer core (Prastowo, 2001; Prastowo & Bijaksana, 2002).

For all generated magnetic models used in the current study, the computed toroidal flow gave a rasionable range of values for the core speed, that is, U_h = 6.9–36.6 km/y or U_h = 0.022–0.116 cm/s, very consistent with previous work (Prastowo & Bijaksana, 2002). The results are also in good agreement with field observations of westward drift 0.17^o per year, usually associated with the horizontal motion of the Earth's fluid outer core of about 10.3 km/y (Bloxham & Jackson, 1991; Buffet, 2000), and with predicted estimates from a 3D numerical simulation on the core's velocity near the CMB that might achieve a maximum speed of 0.1 cm/s (Glatzmaier & Roberts, 1995).

In addition to estimates of the core speed above, we here provide analytical arguments proposed to support for the dipole approximation as the dominant field in the spherical harmonic series of the internally-driven geomagnetic field. We first calculated the contribution of the axial dipole term g_1^0 to the magnitude of the radial field near the Earth's surface using Eq. (1) to obtain

$$B_\mathrm{r} = \int_\theta \ 2g_1^0 \cos\theta \,\mathrm{d}\theta = \int_0^{\pi/2} 2g_1^0 \sin\emptyset \,\mathrm{d}\emptyset, \tag{6}$$

where $\emptyset = 90^o - \theta$ is the latitude. Inserting a typical value for the axial term of $g_1^0 \approx -3 \times 10^4$ nT from all generations of the DGRF/IGRF 1990–2000 models into Eq. (6) and then integrating it resulted in $B_\mathrm{r} = 6 \times 10^4$ nT or $B_\mathrm{r} = 60$ μT. In a similar manner, the contribution of the axial term of the quadrupole g_2^0 could also be estimated using Eq. (1) to give

$$B_\mathrm{r} = \int_\theta \ 1.5 \ g_2^0 \ (3\cos^2\theta - 1) \,\mathrm{d}\theta = -\int_0^{\pi/2} 1.5 \ g_2^0 \ (3\sin^2\emptyset - 1) \,\mathrm{d}\emptyset. \tag{7}$$

Inserting a typical value for the axial quadrupole term of $g_2^0 \approx -2 \times 10^3$ nT from all generations of the DGRF/IGRF 1990–2000 models into Eq. (7) and integrating it resulted in $B_\mathrm{r} = 2.4 \times 10^3$ nT or $B_\mathrm{r} = 2.4$ μT. Estimates of the axial components strongly indicated that the quadrupole is relatively small compared to the dipole as it is only about 4% of the dipole field.

While the contribution of the axial dipole term to the radial field is determined using g_1^0, the contribution of two equatorial terms of the dipole is estimated using g_1^1 and h_1^1 as follows,

$$B_\mathrm{r} = -2\int_\theta \ \sin\theta \,\mathrm{d}\theta \times \int_\varphi \{g_1^1 \ \cos\varphi + h_1^1 \sin\varphi\} \,\mathrm{d}\varphi \tag{8}$$

or alternatively written as

$$B_\mathrm{r} = 2\int_0^{\pi/2} \cos\emptyset \,\mathrm{d}\emptyset \times \int_0^{\pi} \{g_1^1 \ \cos\varphi \ \cos\varphi + h_1^1 \sin\varphi\} \,\mathrm{d}\varphi. \tag{9}$$

Substituting relevant values for $\emptyset$ and φ in Eq. (9) and considering that g_1^1 gives no contribution to the radial field, owing to its span along the Greenwich line separating the west and east directions associated with the zero longitude, then we could estimate B_r in Eq. (9) to be

$$B_\mathrm{r} = 2 \times \{0 + 2\mathrm{h}_1^1\} = 4h_1^1. \tag{10}$$

Substituting $h_1^1 \approx 5 \times 10^3$ nT into Eq. (10), then we obtain $B_\mathrm{r} = 2 \times 10^4$ nT or $B_\mathrm{r} = 20$ μT, which is only one third of the axial dipole term.

The estimate values for the dipole case are in good agreement with measured values of the total field obtained from satellites using high-resolution instruments, where the magnitudes of the main field measured at the Earth's surface are from 25 μT (near the magnetic equator) to 65 μT (near both the magnetic poles), as reported at http://www.ngdc.noaa.gov/IAGA/vmod/. The differences in the minimum and maximum intensities of the main field between theoretical analyses provided here and measured values from field observations are due to terms other than the dipole are not, in this study, included in the calculation.

Further estimates of the radial field contributed from the equatorial quadrupole terms gave an estimate of maximum 10% only, relatively compared to that of the equatorial dipole. Using all the relative contributions of axial and equatorial components derived from analytical arguments, we conclude that the dipole field dominates over all the quadrupole and other terms, where the intensity of the dipole may approximate to 95% of the total intensity of the main field. Hence, the dipole field is considered the primary part of the Earth's magnetic field that is believed of internal origin and that all computations regarding the core velocity in this study are valid.

4. Conclusions and remarks

Time evolution of the geomagnetic field can be used to examine the outer core dynamics by noting that on time-scales of decades and in the limit of perfectly conducting liquid outer core, flow advection dominates over diffusion inside the core. In this condition, only the radial field can continue across the CMB and hence possibly be accurately measured using air observations via satellites. Using the 8–12 Generations of the DGRF/IGRF 1990, 1995, 2000 magnetic models, numerical computations on the basis of toroidal flow assumption for the dipole case show that the outer core's velocity is found to be in the range 7–17 km/y, which is on the order of 10 km/y, in good agreement with previous work (Bloxham & Jackson, 1991; Glatzmaier & Roberts, 1995; Buffett, 2000; Prastowo & Bijaksana, 2002). In the absence of convective flow across the CMB (Whaler, 1982), the results in this study indicate that there is no dependence of the outer core's velocity on the magnetic models used.

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